Magnetic Field Effects on the Transport Properties of One-sided Rough Wires

A. García-Martín*, M. Governale† and P. Wölfle*
* *Institut für Theorie der Kondensierten Materie,*
Universität Karlsruhe, P.O. Box 6980, D-76128 Karlsruhe, Germany
† *Institut für Theoretische Festkörperphysik, Universität Karlsruhe, D-76128 Karlsruhe, Germany*
(Dated: February 1, 2008)

We present a detailed numerical analysis of the effect of a magnetic field on the transport properties of a 'small-N' one-sided surface disordered wire. When time reversal symmetry is broken due to a magnetic field B, we find a strong increase with B not only of the localization length ξ but also of the mean free path ℓ caused by boundary states. Despite this, the universal relationship between ℓ and ξ does hold. We also analyze the conductance distribution at the metal-insulator crossover, finding a very good agreement with Random Matrix Theory with two fluctuating channels within the Circular Orthogonal(Unitary) Ensemble in absence(presence) of B.

PACS numbers: 72.15.Rn, 72.10.Fk, 42.25.Dd, 71.30.+h

The physics of disordered media has attracted the attention of scientists from different areas since long ago[1], and it is an up-to-date topic due to its relevance in wave transport, be it in the form of quantum (electrons) or classical (electromagnetic or acustic) waves. Transport in disordered media, in particular in the mesoscopic regime, presents a rich panoply of phenomena, such as *Anderson localization*[2], *Weak localization*[3], and *Universal Conductance Fluctuations*[4], just to mention a few.

When dealing with disordered systems, it is of fundamental importance to know about the full statistical properties of the relevant transport quantities, i.e. the conductance G, the eigenchannels τ and the transmission T_i, T_{ij} and reflection R_i, R_{ij} coefficients. Much is known about statistics of both the transmission[5, 6] and the reflection[7] coefficients, since the full distribution function has been obtained from a Random Matrix Theory (RMT) approach[5, 7] and numerical techniques[6, 7] as well as experimentally in the microwave regime[8].

Several approaches have been considered to analyze the effect of disorder on the transport properties, ranging from analytical techniques using RMT[9, 10, 11, 12] to numerical ones[13, 14]. Most of them consider that the disorder is inside the material where the wave travels through (bulk disorder). However, the recent developments in nanotechnology open the way to new sources of scattering. In particular, as system sizes are shrunk down to the nanometer scale, the surface-to-volume ratio becomes larger, and surface effects (for example disorder) become very important. This is more relevant in the case of few-modes quantum wires, where the effect of surface disorder (roughness) can dominate their transport properties. In addtion, due to improvements in fabrication techniques bulk disorder is almost absent. Many statistical properties of these surface disordered wires have been shown to be in very good agreement with those predicted by RMT[6, 7, 14, 15, 16].

The consequences of time reversal symmetry breaking in disordered media, such as the doubling of the localization length as a consequence of the phase mismatch of time reversed paths, are conceptually known since long ago (see for instance [1, 10]). Nevertheless, a fundamental question was still not answered: how does the transition take place? This issue has recently been addressed, both theoretically[17] and numerically, by means of an Anderson model (bulk disorder)[18], finding that the expected doubling of the localization length takes place in a rather smooth way, while the mean free path remains constant.

Another conceptual issue concerns the shape of the full conductance distribution $P(G)$ in the diffusion-localization transition. It is known that $P(G)$ evolves from Gaussian, in the diffusive regime, to log-normal in the localization regime. It has been recently predicted[12] that, in the crossover regime, $P(G)$ exhibits a highly non trivial shape; the subsequent numerical analysis revealed a direct relation between the number of eigenchannels that actually describe the distribution and its universal characteristics[14].

In this paper we analyze the consequences that the application of a magnetic field has on the transport properties of a one-sided surface disordered wire (see Fig. 1). We show that breaking time reversal symmetry leads to drastic effects on the electron propagation through rough wires in the small-number-of-channels regime. The dependence of the mean free path ℓ and localization length ξ on the magnetic field strength is studied by means of an analysis of $\langle 1/G\rangle$ and $\langle \ln G\rangle$ as a function of both the length and the magnetic field strength. This analysis reveals a peculiar behavior of this two relevant length scales: ξ increases, but much more than the expected factor of two[17, 18], while at the same time ℓ, which is expected to remain constant (for the low fields considered), also increases. We find that the latter phenomenon is related to the precursors of the edge states (we are far away from the quantum Hall regime), which propagate almost freely due to the specific type of disorder. We show that the unexpected large increase of ξ does not contradict the predictions of RMT when the anomalous behavior of

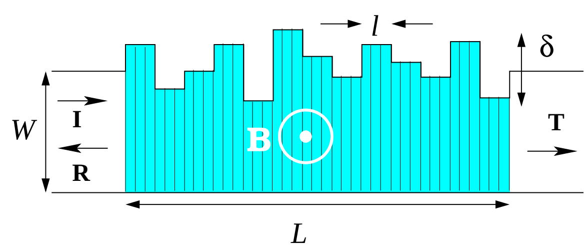

FIG. 1: Schematic view of a nanowire with one-side surface disorder.

ℓ is taken into account. The validity of RMT is confirmed by the analysis of the numerical conductance distribution $P(G)$ in the vicinity of the diffusion-localization transition. This analysis shows that the universal features of the distribution are not affected by the lifting of time reversal invariance.

The system which we are dealing with (depicted schematically in Fig. 1) is made of two semi-infinite two-dimensional clean leads of width W and a surface-disordered region of length L in between. The corrugated region is composed of slices of length ℓ laterally bounded by hard walls, whose random width is uniformly distributed in the interval $[W-\delta, W+\delta]$ around the mean value W. We will take $W/\lambda = 2.6$ (λ being the electron Fermi wavelength), which allows five propagating channels in the clean part $N = 5$, and a disorder strength δ so that $W/\delta = 7$[19]. We consider *One-sided Surface Disorder* (OSD), i.e., the roughness is located only on one surface. In order to study how the presence or absence of time reversal symmetry affects the transport, a magnetic field B perpendicular to the plane of the wire is considered.

The conductance (in units of $2e^2/h$) of the total system is simply given by $G = \mathrm{trace}\{tt^\dagger\}$. The transmission matrix t is calculated for each realization of the disorder by a generalized scattering matrix technique[15, 20] where the magnetic field has been introduced in the formalism as in the method of Ref.[21]. This consists in choosing a gauge, in which only the component of the vector potential along the transverse direction is non zero; the system is then divided in discretization slices (separated by thin lines in Fig. 1), and a constant vector potential is assigned to each slice. To minimize errors due to the discretization, the slices have to be chosen so that they contain less than a flux quantum, furthermore convergence can be checked by increasing the number of them. Then, in order to obtain the statistical features introduced by the disorder, we perform configuration averages $\langle \cdots \rangle$. To calculate the distributions we average over 10000 different configurations of the disorder.

In the upper panel of Fig 2 we plot $\langle \ln G \rangle$ versus L/W for four different values B. The magnetic field strength is given by the quantity $\Phi_\xi = \xi_0 W B$ that measures the

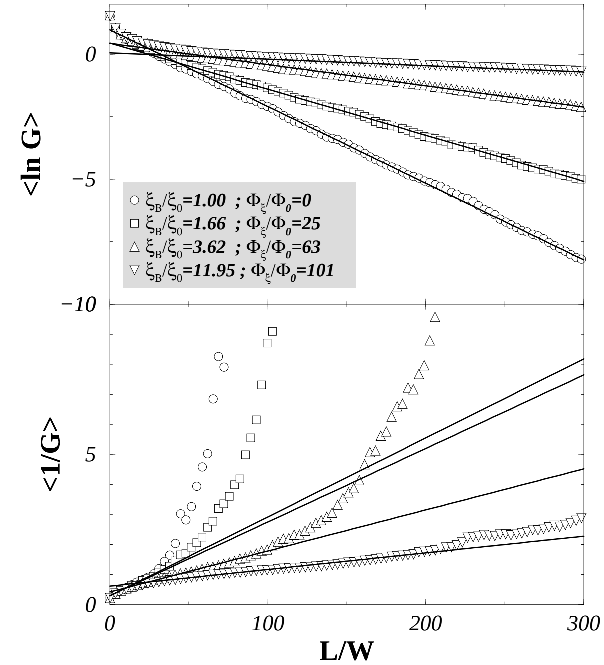

FIG. 2: Logarithm of the conductance (upper panel) and Resistance (lower panel) as a function of the length of the disordered region for different values of the magnetic field (symbols). The lines indicates the best linear fits (obtained in the localization regime for the logarithm of the conductance, and in the diffusive regime for the resistance), which are used to extract respectively the localization length, and the mean free path.

amount of magnetic flux trapped in a portion of wire with a length equal to the localization length at zero field (ξ_0). Throughout this work all magnetic fluxes are given in units of the flux quantum Φ_0.

The localization length ξ_B for the different values of B is obtained from the linear part of the plot since $\langle \ln G \rangle \propto -L/\xi_B$. The value of ξ_B evolves as expected, the higher B the larger ξ_B, but instead of saturating to the expected value $\xi_B = 2\xi_0$[10, 11, 17, 18] when time reversal invariance is broken, for OSD this does not occur giving rise to values $\xi_B \gg 2\xi_0$.

The origin of this lies in the fact that, whereas usually ℓ is assumed to be constant for the small values of B that are considered (see, for example, the results for a two-dimensional Anderson model with on-site disorder of Ref. [18]), in wires with OSD one has to take the magnetic field dependence of ℓ into account. This effect can be seen in lower panel of Fig. 2, where the average resistance $\langle 1/G \rangle$ is plotted as a function of L/W. By virtue of $\langle 1/G \rangle \approx 1/N + L/(N\ell)$ (as long as $\ell < L < \xi$) the best linear fit in the diffusive regime gives ℓ_B.

The general relationship between mean free path and localization length predicted by RMT from general sym-

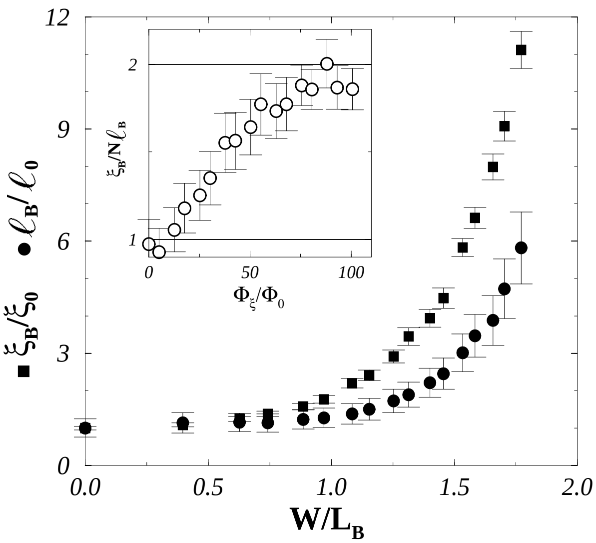

FIG. 3: Magnetic dependence of the mean free path and of the localization length. Both the mean free path and the localization length are normalized to their zero-field value; the magnetic field is in units of $W/L_{\rm B}$, with $L_{\rm B} = \sqrt{\hbar/eB}$, being the magnetic length. Inset: localization length divided by the number of modes and the mean free path; this quantity reaches the value 2 for fully broken time reversal symmetry.

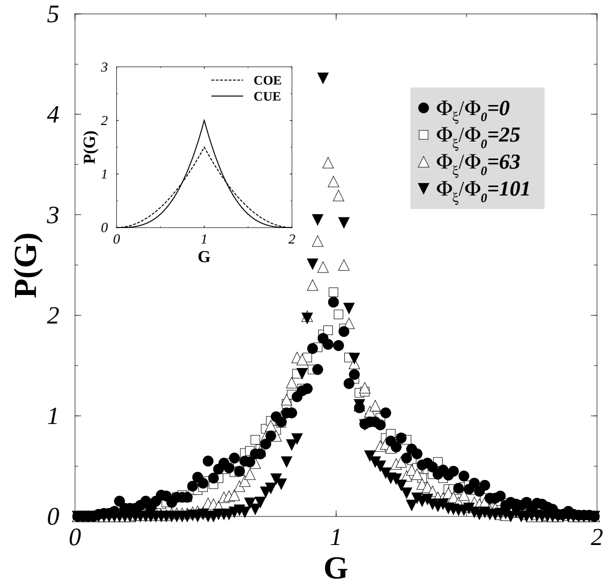

FIG. 4: Conductance distribution close to the diffusion-localization transition for different strength of the magnetic field. In the inset the results from RMT for the maximum entropy Ansatz are shown. The distributions exhibit an increase of the cusp height as well as a decrease of their width with increasing magnetic field.

metry considerations can be written as

$$\xi_{\rm B} = \beta N \ell_{\rm B}, \tag{1}$$

with β being 1 in the presence of time reversal symmetry, and 2 when it is broken (by the magnetic field). In Eq. (1) the magnetic field dependence of the mean free path has to be accounted for.

Eq. (1), predicts a transition from $\beta = 1$ to $\beta = 2$, as soon as time reversal symmetry is removed, i.e. when the magnetic flux comprised in a section of wire of length of the order of ξ_0 is about one flux quantum. As shown in Fig. 3, both ξ_B and ℓ_B increase smoothly when B increases and in the same qualitative way. In fact, the tremendous increase of ℓ_B, allows the possibility to have huge values of ξ_B without violating Eq. (1) in any case (see inset in Fig. 3). The huge increase of these two quantities gives rise to the actual possibility of changing the length of the diffusive regime with a magnetic field. The reason for that lays in the presence of one clean surface. In fact, even for really small B, the developing edge states may propagate along the clean surface suffering almost no scattering. We have checked that this is indeed the case by doing the same analysis for a wire with *two-sided surface disorder*, with uncorrelated disorder of equal strength on both surfaces (not shown here). In the absence of the clean surface, the non fully developed edge states still suffer from scattering, and no appreciable variation of the mean free path is found. It is important to stress once more that the magnetic fields that give rise to the aforementioned effect are small (cyclotron radius much bigger than wire width). Thus, we are *far away* from the quantum Hall regime, where it is natural that edge states propagate freely without suffering any scattering. We note in passing that the magnetoresistance of these one-sided rough wires may offer interesting technical possibilities.

It is worth to mention that our numerical simulations show that the lifting of time reversal invariance is a gradual process, with $\beta = 2$ reached for around 100 flux quanta per localization length times system width, as it is shown in the inset of Fig. 3. This result is in perfect agreement with the values presented in Ref. [18] even though several characteristics of the transport depend on the kind of disorder. We also see no evidence of the 'two-scale localization' predicted in Ref. [17] and not found in Ref. [18].

Now we go beyond the first moments of the distribution, and devote our attention to the entire distribution function $P(G)$ at the transition point. It has been recently reported[14] that $P(G)$ contains some universal characteristics that depend only on the value of $\langle G \rangle$, and in particular, that the transition point corresponds to a transition in the number of active eigenchannels. We would like to address these issues, when the action of the magnetic field eliminates time reversal invariance, by analyzing $P(G)$ in the vicinity of the diffusion-localization transition. Let us focus on the characteristic features of $P(G)$ just before the localization onset, more explicitly

when $\langle G\rangle = 1$. At this value of the mean conductance the distribution has a well define cusp at $G = 1$. In Fig. 4 we show that $P(G)$ for $\langle G\rangle = 1$ shrinks with increasing B and at the same time the cusp becomes higher and more pronounced. This confirms that general characteristics of $P(G)$ at the transition point actually does not depend on the time reversal invariance conditions.

It has been shown that the RMT picture[22] within the maximum entropy Ansatz (this leads to the Circular Orthogonal Ensemble (COE) when time reversal symmetry is present to the Circular Unitary Ensemble (CUE) when it is broken) gives an almost quantitative description of the distributions when time reversal symmetry is preserved[14]. Following the ideas of Ref. [14] we obtain $P(G)$ for two active eigenchannels (Eq. 2):

$$
\begin{aligned}
P(G,\langle G\rangle) \propto \quad & \frac{2}{3}\left(1-\frac{G}{2}\right)^{3}\left(\frac{G}{2}\right)^{2\alpha}\times \\
& {}_2F_1\left(\frac{3}{2},-\alpha,\frac{5}{2},\left(\frac{2}{G}-1\right)^{2}\right)\Theta(G-1) \\
+ \quad & \frac{\sqrt{\pi}}{2}\left(\frac{G}{2}\right)^{2\alpha+3}\frac{\Gamma(1+\alpha)}{\Gamma(\frac{5}{2}+\alpha)}\Theta(1-G). \quad (2)
\end{aligned}
$$

where

$$
\alpha = \frac{4(\langle G\rangle - 1)}{2-\langle G\rangle}\ , \qquad (3)
$$

depends only on the value of the mean conductance[23].

This distribution is shown in in the insets of Fig. 4, together with Eq. 3 from Ref. [14] for COE. The qualitative agreement with the numerical data is striking.

In conclusion, we have presented extensive numerical simulations for nanowires with one-sided surface disorder in the presence of magnetic fields. The transition between the orthogonal and unitary case has been demonstrated to happen smoothly. Although our model of disorder exhibits a large increase of the mean free path leading to giant wave delocalization, the relationship between mean free path and localization length predicted by RMT is fulfilled. The behavior of both mean free path and localization length suggest the possibility of manipulating the diffusion and localization onsets by applying a magnetic field. The conductance distribution around the metal-insulator transition has confirmed its universal nature, being very well described by RMT with the maximum entropy Ansatz and two fluctuating channels.

We are gratefully indebted to F. Evers, A.D. Mirlin, J.J. Sáenz and A. Rosch for valuable discussions. AGM acknowledges support from the Emmy-Noether program of the DFG under Grant No. Bu 1107/2-1. This work was partially supported by the DFG Center for Functional Nanostructures at the University of Karlsruhe.

[1] *"Diffuse Waves in Complex Media"*, J.P. Fouque ed., NATO-ASI Series C **531**, (Kluwer, Dordretch, 1999); P. Sheng *"Introduction to Wave Scattering, Localization and Mesoscopic Phenomena"*, Academic Press, N.Y. (1995); B.L. Altshuler, P.A. Lee and R.A. Webb (eds), *Mesoscopic Phenomena in solids* (North-Holland, Amsterdam, 1991); M. Nieto-Vesperinas and J.C. Dainty eds., *"Scattering in Volumes and Surfaces"*, (North Holland, Amsterdam, 1990).

[2] P. W. Anderson, Phys. Rev. **109** 1492 (1958).

[3] E. Abrahams,P. W. Anderson, D. C. Licciardello, and T. V. Ramakrishnan, Phys. Rev. Lett. **42**, 673 (1979).

[4] C.P. Umbach, S. Washburn, R.B. Laibowitz, and R.A. Webb, Phys. Rev. B 30, 4048 (1984); P.A. Lee, A.D. Stone, and H. Fukuyama, 35, 1039 (1987).

[5] S.A. van Langen, P.W. Brouwer and C.W.J. Beenakker, Phys. Rev. E **53**, 1344 (1996).

[6] A. García-Martín, J.A. Torres, J.J. Sáenz and M. Nieto-Vesperinas, Phys. Rev. Lett. **80**, 4165 (1998).

[7] A. García-Martín, T. López-Ciudad, J.J. Sáenz and M. Nieto-Vesperinas, Phys. Rev. Lett. **81**, 329 (1998).

[8] M. Stoytchev and A.Z. Genack, Phys. Rev. Lett. **79**, 309 (1997).

[9] O.N. Dorokhov, JETP Lett. **36**, 318 (1982); P.A. Mello, P. Pereyra, and N. Kumar, Ann. Phys. (N.Y.) **181**, 290 (1988).

[10] C.W.J. Beenakker, Rev. Mod. Phys. **69**, 731 (1997).

[11] J.-L. Pichard in *Quantum Coherence in Mesoscopic Systems*, edited by B. Kramer, NATO ASI Series B, **254** (Plenum, New York), -p. 369 (1991).

[12] K. A. Muttalib and P. Wölfle, Phys. Rev. Lett. , **83**, 3013 (1999).

[13] B. Jovanović, and Z. Wang, Phys. Rev. Lett. **81**, 2767 (1998); P. Markoš, *ibid.* **83**, 588 (1999); S. Cho, Phys. Rev. B **55**, 1637 (1997); V. Plerou and Z. Wang, *ibid.* **58**, 1967 (1998); X. Wang, Q. Li and C.M. Soukoulis, *ibid.* **58**, 3576 (1998); M. Rühländer, P. Markoš and C. M. Soukoulis, *ibid.* **64**, 212202 (2001);

[14] A. García-Martín, and J.J. Sáenz, Phys. Rev. Lett. **87**, 116603 (2001).

[15] A. García-Martín, J.A. Torres, J.J. Sáenz and M. Nieto-Vesperinas, Appl. Phys. Lett. **71**, 1912 (1997).

[16] J.A. Sánchez-Gil *et al.*, Phys. Rev. Lett. **80**, 948 (1998); Phys. Rev. B **59**, 5915 (1999); A. García-Martín *et al.*, Phys. Rev. Lett. **84**, 3578 (2000); Phys. Rev. B **62**, 9386 (2000).

[17] A. V. Kolesnikov, and K. .B. Efetov, Phys. Rev. Lett. **83**, 3689 (1999).

[18] H. Schomerus, and C. W. J. Beenakker, Phys. Rev. Lett. **84**, 3927 (2000).

[19] We have verified that changing the values of the parameters, in the small-N, weak scattering regime, does not affect the physical picture described in this work.

[20] A. Weisshaar, J. Lary, S. M. Goodnick and V.K. Tripathi, J. Appl. Phys. **70**, 355 (1991); J.A. Torres and J.J. Sáenz, unpublished; J.A. Torres, Ph. D. Thesis, Universidad Autónoma de Madrid (1997).

[21] M. Governale and D. Boese, Appl. Phys. Lett. **77**, 3215 (2000).

[22] H.U. Baranger and P.A. Mello, Phys. Rev. Lett. **73**, 142 (1994); R.A. Jalabert, J.L. Pichard and C.W.J.

Beenakker, Europhys. Lett. **27**, 255 (1994).
[23] This expression is valid for $2/3 < \langle G \rangle \lesssim 5/4$, i.e. when the distribution can be described by two fluctuating channels.